3/6/18 "Too many of us are not living our dreams because we are living our fears."-Les Brown

HW: "2017 Calc L26 Related Rates" page 12 (Top question)
Test 2 on Friday 3/9

AIM: More Related Rates

Warm Up:

Let $f$ be the function given by $f(x) = 2e^{4x^2}$. For what value of $x$ is the slope of the line tangent to the graph of $f$ at $(x, f(x))$ equal to 3 ?

derivative

(A) 0.168 (B) 0.276 (C) 0.318 (D) 0.342 (E) 0.551

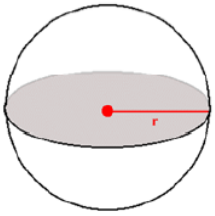
Derivative = 3

$Y_1$ $Y_2$

2nd Trace 5

Sphere

1) Gas is being pumped into a spherical balloon at a rate of 5 $ft^3/min$. Find the rate at which the radius is changing when the diameter is 18 inches.



Know:

$\frac{dV}{dt} = 5\ \frac{ft^3}{min}$

Need:

$\frac{dr}{dt}$

$d = 18in$

$r = 9in = \frac{3}{4}ft$

Equation:

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$5 = 4\pi\left(\frac{3}{4}\right)^2 \frac{dr}{dt}$

$5 = 4\pi\left(\frac{9}{16}\right) \frac{dr}{dt}$

$\frac{5}{\pi} = \frac{9}{4}\pi \frac{dr}{dt}$

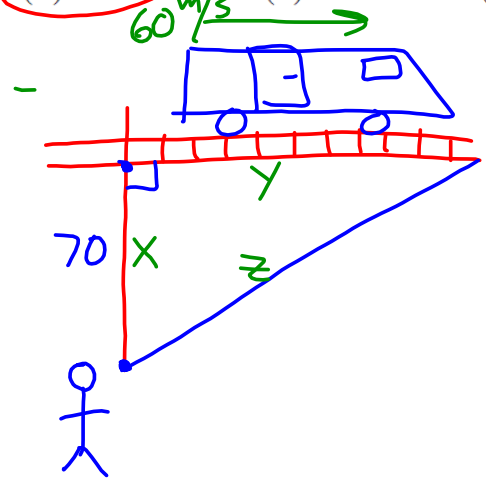
$\left(\frac{4}{9}\right)\frac{5}{\pi} = \frac{9}{4}\frac{dr}{dt}\left(\frac{4}{9}\right)$

$\frac{20}{9\pi} = \frac{dr}{dt}$

$\frac{20}{9\pi}\ \frac{ft}{min}$

1. A railroad track and a road cross at right angles. An observer stands on the road 70 meters south of the crossing and watches an eastbound train traveling at 60 meters per second. At how many meters per second is the train moving away from the observer 4 seconds after it passes through the intersection?

(A) 57.60 (B) 57.88 (C) 59.20 (D) 60.00 (E) 67.40




Know:

$x = 70$ $\frac{dx}{dt} = 0$

$y = 240$ $\frac{dy}{dt} = 60$

$z = 250$ $\frac{dz}{dt} =$

$(60)(4) = 240$

$x^2 + y^2 = z^2$

$70^2 + 240^2 = z^2$

$4900 + 57600 = z^2$

$62500 = z^2$

$250 = z$

$x^2 + y^2 = z^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$2(70)(0) + 2(240)(60) = 2(250)\frac{dz}{dt}$

$0 + 28800 = 500\frac{dz}{dt}$

$\frac{28800}{500} = \frac{500\frac{dz}{dt}}{500}$

$57.6 = \frac{dz}{dt}$

57.6 m/sec

2. A Stone thrown into a pond produces a circular ripple which expands from the point of impact. If the radius of the ripple increases at the rate of 1.5m/sec, how fast is the disturbed area growing when the radius is 8m?

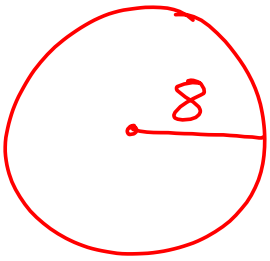
Know:

$r = 8m$

$\frac{dr}{dt} = 1.5 m/s$

Need:

$\frac{dA}{dt}$



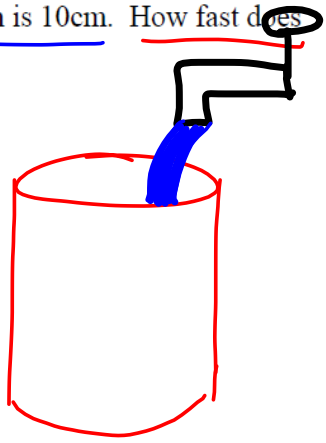

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\frac{dA}{dt} = 2(8)(1.5)\pi$

$\frac{dA}{dt} = 24\pi \ m^2/sec$

3. A can is being filled with water at a rate of $50 cm^3/min$. The radius of the can is 10cm. How fast does the height of the water in the can rise?



Know:

$\frac{dV}{dt} = 50 cm^3/min$

$r = 10 cm$

$\frac{dr}{dt} = 0$ (can stays same size)

Need:

$\frac{dh}{dt} = ?$

$V = \pi r^2 h$

$\frac{dV}{dt} = 2\pi r \frac{dr}{dt} h + \pi r^2 \frac{dh}{dt}$

$50 = 2\pi(10)(0)h + \pi 10^2 \frac{dh}{dt}$

don't need

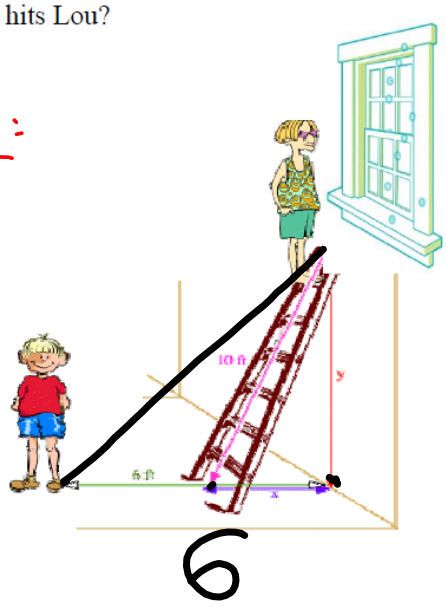
$\frac{50}{100\pi} = 0 + \frac{100\pi \frac{dh}{dt}}{100\pi}$

$\frac{1}{2\pi} = \frac{dh}{dt}$

$\frac{1}{2\pi}$ cm/min

Triangles

Joey is perched precariously the top of a 10-foot ladder leaning against the back wall of an apartment building (spying on an enemy of his) when it starts to slide down the wall at a rate of 4 ft per minute. Joey's accomplice, Lou, is standing on the ground 6 ft away from the wall. How fast is the base of the ladder moving when it hits Lou?




Know:

$x = 6 \quad \frac{dx}{dt} = ?$

$y = 8$ (Pythag. theorem) $\quad \frac{dy}{dt} = -4 \text{ ft/min}$

$z = 10 \quad \frac{dz}{dt} = 0$ (ladder)

Need:

$\frac{dx}{dt}$

$x^2 + y^2 = z^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$2(6)\left(\frac{dx}{dt}\right) + 2(8)(-4) = 2(10)(0)$

$12\frac{dx}{dt} + -64 = 0$

$\frac{12\frac{dx}{dt}}{12} = \frac{64}{12}$

$\frac{dx}{dt} = \frac{16}{3} \text{ ft/min}$